CONWAY POLYNOMIALS OF $Th(p,q)$ AND $Th(q,p)$: A PROOF OF TAKEMURA'S CONJECTURE

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Abstract. Takemura conjectured that the Conway polynomials $\sum a_\ell z^\ell$ of the Turk's head link $Th(p,q)$ and $\sum b_\ell z^\ell$ of $Th(q,p)$ satisfy $a_\ell = b_\ell$ for $\ell \equiv 0,1 \pmod 4$ and $a_\ell = -b_\ell$ for $\ell \equiv 2,3 \pmod 4$, and proved it for $q = 2,3$. We prove it for all $p, q \geq 1$, knots and links. Equivalently, $\nabla_{Th(q,p)}(z) = c\,\nabla_{Th(p,q)}(iz)$ with $c = 1$ if $\gcd(p,q)$ is odd and $c = -i$ if it is even. The proof combines a product formula $\nabla_{Th(p,q)}(z)^2 = \prod_{j=1}^{p-1}\prod_{k=1}^{q-1}\left(z^2 + 2\cos\frac{2\pi k}{q} - 2\cos\frac{2\pi j}{p}\right)$, which comes from the reduced Burau representation, with the leading coefficients, read off the Seifert surface of the closed braid.

1. Result

For $p \geq 1$ let $A_p = \sigma_1\sigma_2^{-1}\sigma_3\sigma_4^{-1}\cdots\sigma_{p-1}^{\pm 1} \in B_p$ (A_1 is the trivial braid). The Turk's head link $Th(p,q)$ is the closure of A_p^q; it has $\mu = \gcd(p,q)$ components [DS, §2], and we orient it as a closed braid, all strands running in the same direction. For an oriented link L let $\nabla_L(z) \in \mathbb{Z}[z]$ be its Conway polynomial ($\nabla_{\text{unknot}} = 1$, $\nabla_{L_+} - \nabla_{L_-} = z\nabla_{L_0}$); only powers z^ℓ with $\ell \equiv \mu - 1 \pmod 2$ occur. We write $\Delta_L(t) = \nabla_L(t^{1/2} - t^{-1/2})$ for the Conway-normalised Alexander polynomial.

Takemura [Tak16, Thm. 1.1] proved for $m \geq 1$ and $n = 2,3$ that, writing $\nabla_{Th(m,n)} = \sum a_\ell z^\ell$ and $\nabla_{Th(n,m)} = \sum b_\ell z^\ell$,

$$a_\ell = b_\ell \ \text{ for } \ell \equiv 0,1 \pmod 4, \qquad a_\ell = -b_\ell \ \text{ for } \ell \equiv 2,3 \pmod 4, \tag{1}$$

and conjectured (1) for all $m \geq 1$, $n \geq 2$ [Tak16, Conj. 5.1]. Di Prisa and Şavk record the conjecture next to their Problem E [DS, §6.4].

Theorem 1. (1) *holds for all $p, q \geq 1$, i.e. for $\nabla_{Th(p,q)} = \sum a_\ell z^\ell$ and $\nabla_{Th(q,p)} = \sum b_\ell z^\ell$.*

Date: 19 October 2026 (first version dated 5 October 2026, on Zenodo since 6 October 2026).

2020 *Mathematics Subject Classification.* Primary 57K10; Secondary 57K14.

This work was developed in collaboration with the AI system Claude Code (Anthropic): the choice of problem, the direction of the work and the computing hardware are due to the author (a sculptor); the formalisation, the algorithms and the proofs were developed by Claude Code (Anthropic) working with the author. The collaboration is stated as a fact, not a disclaimer; details are in the section on tool use at the end.

Here $i = \sqrt{-1}$. Since $i^\ell = (-1)^{\ell/2}$ for even ℓ and $-i \cdot i^\ell = (-1)^{(\ell-1)/2}$ for odd ℓ, and since $a_\ell = b_\ell = 0$ unless $\ell \equiv \mu - 1 \pmod 2$, Theorem 1 is equivalent to

$$\nabla_{Th(q,p)}(z) = c_\mu \, \nabla_{Th(p,q)}(iz), \qquad c_\mu = \begin{cases} 1, & \mu \text{ odd}, \\ -i, & \mu \text{ even}. \end{cases} \tag{2}$$

Mirroring both links replaces $\nabla(z)$ by $\nabla(-z)$ on both sides, so (1) and (2) do not depend on the handedness convention for A_p. If $p = 1$ or $q = 1$ both links are unknots and (1) is trivial; we assume $p, q \geq 2$. For $p = q$, (1) says that $a_\ell = 0$ for all $\ell \equiv 2, 3 \pmod 4$.

Remark 2 (What we have read). We have read [Tak16] (in its accepted manuscript, Kobe University Repository, hdl:20.500.14094/90003785), [DS] and Takemura's doctoral thesis [TakD] (cited in [Sa] as "Tech. Report 7116"). The thesis proves the cases $n = 2, 3$, states "We can not prove this Theorem for $n \geq 4$ yet", and repeats the conjecture; its Section 6 gives a Seifert matrix of $Th(m, n)$ of the block form described in Section 3 [TakD, Lemma 6.1]. Takemura's article [Tak18] is quoted in [DS, §4.6] for a recursive determinant formula for $\Delta_{Th(p,q)}$ and a divisibility theorem; we have not read it. A search of Semantic Scholar on 5 October 2026 found [DS] and [Tak18] as the only works citing [Tak16], and [DS] records it as a conjecture, in its section on open problems (§6.4).

What is known about the product formula: Blackwell, Das, Mayer, Moyar, Quraishi and Stees [BDMMQS, Thm. 3.5] write the Alexander polynomial of a spiral knot, and by their Section 5 of a spiral link, as a product over the non-trivial q-th roots of unity of a recursively defined polynomial C_{p-1}; the closed form of the factors used below is [H, Thm. 2].

2. The product formula

Put $u_k = 2\cos(2\pi k/q)$, $v_j = 2\cos(2\pi j/p)$ and

$$P_{p,q}(z) := \prod_{j=1}^{p-1} \prod_{k=1}^{q-1} \left(z^2 + u_k - v_j\right).$$

Its coefficients are rational integers: they are algebraic integers, and the Galois group of $\mathbb{Q}(e^{2\pi i/pq})$ permutes the u_k among themselves and the v_j among themselves.

Lemma 3. *For $p, q \geq 2$, $\nabla_{Th(p,q)}(z)^2 = P_{p,q}(z)$.*

The proof uses two cited facts.

(B) Burau's theorem. For a braid $\beta \in B_p$ and S the reduced Burau matrix, obtained from Birman's matrices (3-26) by deleting the last row and column [Bir74, p. 122], $(1 + t + \cdots + t^{p-1})\,\tilde{\nabla}_{\hat{\beta}}(t) = \det\left(S(\beta) - I\right)$ [Bir74, Thm. 3.11, (3-28)], where $\tilde{\nabla}$ is the reduced (one-variable) Alexander

polynomial of the closure; by Birman's footnote 7 on p. 120 this holds for links of any number of components.

(S) Two standard facts, used without re-proof: (S1) $\tilde{\nabla}_L(t)$ agrees, up to a unit $\pm t^k$, $k \in \frac{1}{2}\mathbb{Z}$, with $\det(t^{1/2}V - t^{-1/2}V^{\mathsf{T}})$ for any Seifert matrix V of L; (S2) there is a sign $\eta \in \{\pm 1\}$, fixed by the conventions and the same for all oriented links, such that $\nabla_L(t^{1/2} - t^{-1/2}) = \eta^n \det(t^{1/2}V - t^{-1/2}V^{\mathsf{T}})$ for every Seifert matrix V of L of size n. (SageMath computes ∇ through a Seifert matrix; the checks in Section 5 agree with (S1) and (S2).)

(F) The factorisation of [H]. With $m = \lfloor (p-1)/2 \rfloor$ and $c_j = 4\cos^2(\pi j/p) = 2 + v_j$,

$$\det\big(I - x\,S(A_p)\big) = (1+xt)^{[p \text{ even}]}\, t^{-m} \prod_{j=1}^{m} \big(x\,t^2 + (x^2 - c_j x + 1)t + x\big)$$

for $p \geq 3$ [H, Thm. 2]; for $p = 2$, $S(A_2) = (-t)$ and the same formula holds with $m = 0$.

Proof of Lemma 3. Let $\zeta = e^{2\pi i/q}$ and $z = t^{1/2} - t^{-1/2}$, so $t + t^{-1} = z^2 + 2$. Since $\det(I - S^q) = \prod_{k=0}^{q-1} \det(I - \zeta^k S)$ and the $k = 0$ factor is $t^{-m}(1 + t + \cdots + t^{p-1})$ [H, proof of Cor. 4], (B), (S1), (S2) and (F) give, up to a unit $\pm t^{\nu/2}$, $\nu \in \mathbb{Z}$ (the scalars ζ^k below multiply to ± 1 over $1 \leq k \leq q-1$),

$$\nabla_{Th(p,q)}\big(t^{1/2} - t^{-1/2}\big) \;\doteq\; \prod_{k=1}^{q-1} \Big[(1+\zeta^k t)^{[p \text{ even}]} \prod_{j=1}^{m} Q_j(t, \zeta^k) \Big],$$

where $Q_j(t,x) = x\,t^2 + (x^2 - c_j x + 1)t + x$. For $x = \zeta^k$ we have $x + x^{-1} = u_k$, hence $Q_j(t,x) = xt\,(t + t^{-1} + u_k - c_j) = xt\,(z^2 + u_k - v_j)$. For even p the linear factors pair up: for $k \neq q-k$, $(1+\zeta^k t)(1+\zeta^{-k}t) = t\,(t + t^{-1} + u_k) = t\,(z^2 + u_k - v_{p/2})$, because $v_{p/2} = -2$; for even q the unpaired factor $k = q/2$ is $1 - t = -t^{1/2}z$. So $\nabla_{Th(p,q)}(z) \doteq G(z)$ with

$$G(z) = \prod_{k=1}^{q-1} \prod_{j=1}^{m} \big(z^2 + u_k - v_j\big) \cdot \Big[\prod_{k=1}^{\lfloor (q-1)/2 \rfloor} \big(z^2 + u_k - v_{p/2}\big) \Big]^{[p \text{ even}]} \cdot z^{[p,q \text{ even}]}.$$

Now compare G^2 with $P_{p,q}$. Since $v_j = v_{p-j}$, the indices j and $p-j$ give equal factors, and $j = p/2$ (for even p) is the only fixed point; likewise $u_k = u_{q-k}$ with fixed point $k = q/2$ (for even q). Hence

$$P_{p,q} = \prod_{k=1}^{q-1} \prod_{j=1}^{m} (z^2 + u_k - v_j)^2 \cdot \Big[\prod_{k=1}^{q-1} (z^2 + u_k - v_{p/2}) \Big]^{[p \text{ even}]},$$

and the last product equals $\prod_{k \leq \lfloor (q-1)/2 \rfloor} (z^2 + u_k - v_{p/2})^2$, times $z^2 + u_{q/2} - v_{p/2} = z^2$ when q is also even. Thus $P_{p,q} = G^2$.

It remains to remove the unit. Write $s = t^{1/2}$; then $\nabla(s - s^{-1}) = \pm s^k G(s - s^{-1})$ for some $k \in \mathbb{Z}$. Both $\nabla(s - s^{-1})$ and $G(s - s^{-1})$ are invariant under

$s \mapsto -s^{-1}$, so $s^k = (-1)^k s^{-k}$, which forces $k = 0$ because $G \neq 0$. Hence $\nabla_{Th(p,q)} = \pm G$ and $\nabla^2_{Th(p,q)} = P_{p,q}$. □

Remark 4 (Determinants). At $t = -1$, i.e. $z = 2i$, $|\nabla_L(2i)| = |\Delta_L(-1)| = \det L$, so Lemma 3 gives $\det Th(p,q) = |P_{p,q}(2i)|^{1/2}$, where $-4 + u_k - v_j = -\big(4\sin^2\frac{\pi k}{q} + 4\cos^2\frac{\pi j}{p}\big)$. Let p be even. The factors with j and $p-j$ are equal since $v_j = v_{p-j}$; we write $\cos^2\frac{\pi j}{p} = \sin^2\frac{\pi(p/2-j)}{p}$, and the factors with $j = p/2$ contribute $\prod_{k=1}^{q-1} 4\sin^2\frac{\pi k}{q} = q^2$ to $|P_{p,q}(2i)|$; hence

$$\det Th(p,q) = q \prod_{k=1}^{q-1} \prod_{j=1}^{p/2-1} \Big(4\sin^2\frac{\pi k}{q} + 4\sin^2\frac{\pi j}{p}\Big).$$

This is the determinant formula of Dowdall, Mattman, Meek and Solis for even $p \geq 4$, $q \geq 3$, as recorded in [DS, Thm. H].

3. Leading coefficients

We read the leading coefficient off the Seifert surface of the closed braid $\widehat{A^q_p}$, so that the orientation conventions are the same for $Th(p,q)$ and $Th(q,p)$. Write A^q_p with the letters $\sigma_i^{s_i}$, $s_i = (-1)^{i+1}$, and call σ_1 a positive crossing (the opposite convention is discussed after Lemma 5).

The surface. The Seifert circles of the closed braid are p concentric circles, all oriented the same way. Let F consist of discs $D_1, \dots, D_p$ bounded by them, D_i at height $h = i$, and of one half-twisted band for each crossing; the q bands $B_{i,1}, \dots, B_{i,q}$ of the letters $\sigma_i^{s_i}$ join D_i to D_{i+1} inside the slab $i \leq h \leq i+1$. Then F is a connected Seifert surface with $\chi(F) = p - q(p-1)$, so $H_1(F)$ has rank $N = (p-1)(q-1)$. For each i, $F_i = D_i \cup D_{i+1} \cup \bigcup_k B_{i,k}$ is the surface of Seifert's algorithm for the closure of the 2-braid $\sigma_1^{s_i q}$, and $H_1(F)$ has the basis $\gamma_{i,k}$ ($1 \leq i \leq p-1$, $1 \leq k \leq q-1$), where $\gamma_{i,k} \subset F_i$ runs through $B_{i,k}$ and $B_{i,k+1}$.

The Seifert matrix is block triangular. Let $V(a,b) = \mathrm{lk}(a, b^+)$, b^+ the push-off of b to the positive side of F, and order the basis by i. For $j \geq i+1$ the curves $\gamma_{i,k} \subset \{h \leq i+1\}$ and $\gamma_{j,l} \subset \{h \geq i+1\}$ meet the plane $h = i+1$ at most in D_{i+1}, where F is horizontal. If the positive normal of the discs points up, the push-off $\gamma^+_{j,l}$ can be taken in $\{h > i+1\}$, so $\gamma_{i,k}$ and $\gamma^+_{j,l}$ are separated by a plane and $V(\gamma_{i,k}, \gamma_{j,l}) = 0$; if it points down, the same argument gives $V(\gamma_{j,l}, \gamma_{i,k}) = 0$. Either way V is block triangular, and its diagonal block V_i is the Seifert matrix of F_i in the basis $\gamma_{i,\cdot}$ (the linking numbers of curves on F_i and their push-offs do not see the rest of F). This is the block form of the Seifert matrices in [TakD, Lemma 6.1] and [DS, §4.6].

Lemma 5. *With σ_1 a positive crossing, $\nabla_{Th(p,q)}$ has degree $N = (p-1)(q-1)$ and leading coefficient $(-1)^{(q-1)\lfloor (p-1)/2 \rfloor}$.*

Proof. By (S2), for a Seifert matrix V of size n the polynomial ∇_L has degree at most n, and its coefficient of z^n is $\eta^n \det V$: the coefficient of $t^{n/2}$

in $\det(t^{1/2}V - t^{-1/2}V^{\mathsf{T}})$ is $\det V$, and z^n is the only power of $z = t^{1/2} - t^{-1/2}$ containing $t^{n/2}$, with coefficient 1.

For the closure L_n of σ_1^n ($n \geq 0$) the skein relation at one crossing gives $\nabla_{L_n} = z\nabla_{L_{n-1}} + \nabla_{L_{n-2}}$, with $\nabla_{L_0} = 0$ (two-component unlink) and $\nabla_{L_1} = 1$; so ∇_{L_n} has degree $n-1$ and leading coefficient 1. For the closure L_{-n} of σ_1^{-n} likewise $\nabla_{L_{-n}} = -z\nabla_{L_{-(n-1)}} + \nabla_{L_{-(n-2)}}$, leading coefficient $(-1)^{n-1}$. Since $\partial F_i = L_{s_i q}$ and V_i has size $q-1$, this gives $\eta^{q-1}\det V_i = 1$ for $s_i = 1$ and $= (-1)^{q-1}$ for $s_i = -1$; in particular $\det V_i \neq 0$. Hence $\det V = \prod_i \det V_i \neq 0$, so $\nabla_{Th(p,q)}$ has degree exactly N, and its leading coefficient is

$$\eta^N \det V = \prod_{i=1}^{p-1} \eta^{q-1} \det V_i = \prod_{i:\, s_i = -1} (-1)^{q-1} = (-1)^{(q-1)\lfloor (p-1)/2 \rfloor},$$

as $s_i = -1$ exactly for the $\lfloor (p-1)/2 \rfloor$ even indices $i \leq p-1$. The unknown sign η has cancelled. □

With the opposite convention (σ_1 negative) every s_i changes sign, and the exponent becomes $(q-1)\lceil (p-1)/2 \rceil$; this is the mirror image, and by Section 1 the theorem does not depend on it. The value of c below is also the same with either exponent.

4. Proof of Theorem 1

Let $p, q \geq 2$, $\mu = \gcd(p,q)$, $N = (p-1)(q-1)$. Interchanging the roles of j and k,

$$P_{q,p}(z) = \prod_{k=1}^{q-1}\prod_{j=1}^{p-1} \left(z^2 - u_k + v_j\right) = (-1)^N \prod_{k,j} \left((iz)^2 + u_k - v_j\right) = (-1)^N P_{p,q}(iz).$$

By Lemma 3, $\nabla_{Th(q,p)}(z)^2 = (-1)^N \nabla_{Th(p,q)}(iz)^2$ in $\mathbb{C}[z]$, so $\nabla_{Th(q,p)}(z) = c\,\nabla_{Th(p,q)}(iz)$ for a constant c with $c^2 = (-1)^N$. Comparing coefficients of z^N by Lemma 5 (both leading coefficients are ± 1),

$$c = (-1)^E\, i^{-N}, \qquad E = (p-1)\lfloor (q-1)/2 \rfloor + (q-1)\lfloor (p-1)/2 \rfloor.$$

p, q odd. $E = (p-1)(q-1)$ is even, and $N \equiv 0 \pmod 4$; so $c = 1 = c_\mu$.

p even, q odd (the case p odd, q even is symmetric). $E = (p-1)\frac{q-1}{2} + (q-1)\frac{p-2}{2} = \frac{q-1}{2}(2p-3) \equiv \frac{q-1}{2} \pmod 2$, and since $q-1$ is even and $p-1$ odd, $i^{-N} = \left(i^{q-1}\right)^{-(p-1)} = \left((-1)^{(q-1)/2}\right)^{-(p-1)} = (-1)^{(q-1)/2}$. So $c = 1 = c_\mu$.

$p = 2a$, $q = 2b$. $E = (2a-1)(b-1) + (2b-1)(a-1) \equiv a + b \pmod 2$, and $N = 4ab - 2a - 2b + 1$, so $i^{-N} = -i\,(-1)^{a+b}$. So $c = -i = c_\mu$.

(With $\lceil \cdot \rceil$ in place of $\lfloor \cdot \rfloor$, E changes by $[p \text{ even}](q-1) + [q \text{ even}](p-1)$, which is even in every case.)

In all cases (2) holds, which is Theorem 1. □

Remark 6 (Degree for links). For links the degree of $\Delta_{Th(p,q)}$ (span in t) is $N = (p-1)(q-1) = 2g_3 + \mu - 1$ (Lemma 5); the expression $(p-1)(q-$

$1) + 1 - \gcd(p,q)$ in [DS, §4.6(2)] is $2g_3$ [DS, Thm. E] and agrees with it exactly for knots, where $\mu = 1$; for links the span is $2g_3 + \mu - 1$. For instance $\nabla_{Th(2,2)} = \pm z$ has degree 1.

5. Checks

The proof does not depend on them. Conway polynomials were computed with SageMath's `Link.conway_polynomial()` (Seifert-matrix route, independent of Burau) for the closure of A_p^q.

- (1) holds for all 28 pairs $2 \le p \le q \le 8$ (knots and links); the variant without the sign change fails in all 21 pairs with $p \ne q$.
- All 29 Conway polynomials listed in [Tak16, §1 and §5] are reproduced: 18 exactly, and the 11 with even μ up to $z \mapsto -z$, i.e. our A_p is the mirror of Takemura's convention. This is visible only for even μ: for odd μ the link has an odd number μ of components, and its Conway polynomial contains only even powers of z.
- (2) holds exactly (coefficients in $\mathbb{Q}(i)$) for all 81 pairs $2 \le p, q \le 10$, and the degree and leading coefficient of Lemma 5 hold in all 81; with $c = +i$ instead of $-i$ the identity fails in all 25 pairs with p, q even.
- Lemma 3 exactly (coefficients in cyclotomic fields) for all 64 pairs $2 \le p, q \le 9$; the variant with factors $z^2 - u_k + v_j$ fails in all 56 pairs with $p \ne q$.
- The block matrix V of [DS, §4.6] (diagonal blocks alternately A_q and $-A_q^{\mathsf{T}}$, below them $-A_q$, A_q with -1 on the diagonal and 1 above it) for $2 \le p, q \le 8$ (49 pairs): $\det(t^{1/2}V - t^{-1/2}V^{\mathsf{T}})$ equals SageMath's Conway polynomial exactly for odd μ (33) and up to $z \mapsto -z$ for even μ (16); $\det V = (-1)^{(q-1)\lceil (p-1)/2 \rceil}$, and this is the leading coefficient of that polynomial, in all 49 (in this matrix convention the exponent has $\lceil \cdot \rceil$; cf. the remark after Lemma 5); with all diagonal blocks equal to A_q (no $-A_q^{\mathsf{T}}$) the result is wrong in all pairs with $p \ge 3$ (42 of the 49).
- The value of c in Section 4 equals c_μ for all $2 \le p, q \le 400$ (159 201 pairs), with $\lfloor \cdot \rfloor$ and with $\lceil \cdot \rceil$.

The scripts and their outputs are deposited at doi:10.5281/zenodo.23183920.

Tool use. This work was prepared with substantial use of Claude Code (Anthropic), an agentic AI system based on large language models, working together with the author.

- *Mathematics.* Takemura's conjecture was among the open problems found in a literature search carried out with Claude Code, in the survey [DS, §6.4]; the author decided that this note be written. The proof was worked out by Claude Code under the author's direction.
- *Writing and typesetting.* The text and the LaTeX source were written by Claude Code, in English. The note contains no figures.
- *References.* Literature was searched and sourced with Claude Code. Quotations and statements from [Tak16], [TakD], [DS], [BDMMQS], [Sa] and

[Bir74] were checked against these texts (for [Bir74], against scans of the cited pages); [Tak18] was not read and is cited as in [DS]. The bibliographic data of [Tak16] were checked against Crossref.

- *Code.* The checks of Section 5 were written and run by Claude Code in Python, with SageMath 10.9 (Python 3.12.14). The deposited scripts, at doi:10.5281/zenodo.23183920, are English versions of the working scripts, prepared by Claude Code and run again before deposit, with the results stated in Section 5.
- *Checking.* The draft was read in full by separate Claude Code agents that had not read the working notes, twice on 5 October and once on 19 October 2026, and each later change was checked in a diff read; their findings were checked and incorporated where they held.

The author takes full responsibility for all contents.

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